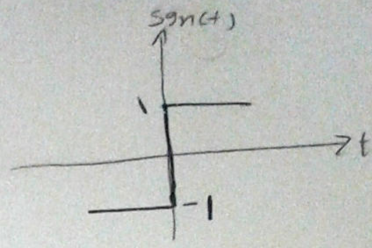
- الاتصالات - Sec 4

(8) Find FT for $g(t) = \mathrm{sgn}(t)$

Sol



$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} \, dt$$

$$= \int_{-\infty}^{0} -1 \, e^{-j2\pi f t} \, dt + \int_{0}^{\infty} 1 \, e^{-j2\pi f t} \, dt$$

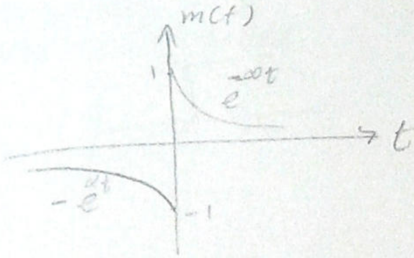
$$= - \frac{e^{-j2\pi f t}}{-j2\pi f} \Big|_{-\infty}^{0} + \frac{e^{-j2\pi f t}}{-j2\pi f} \Big|_{0}^{\infty}$$

$$= \frac{1}{j2\pi f} [e^{0} - e^{\infty}] - \frac{1}{j2\pi f} [e^{-\infty} - e^{0}]$$

$G(f) = \infty$ X

$m(t) \xrightarrow{\text{relation}} \mathrm{sgn}(t)$

$F[\mathrm{sgn}(t)] \longrightarrow M(f)$



$$\mathrm{sgn}(t) = \lim_{\alpha \to 0} m(t)$$

$$\Downarrow$$

$$F[\mathrm{sgn}(t)] = \lim_{\alpha \to 0} M(f)$$

$$m(t) = e^{-\alpha t} u(t) - e^{\alpha t} u(-t)$$

⇒ Using super Position:

$$e^{-\alpha t} u(t) = \frac{1}{\alpha + j2\pi f}$$

$$e^{\alpha t} u(-t) \longrightarrow \frac{1}{\alpha - J 2\pi f}$$

$$M(f) = \frac{1}{\alpha + J 2\pi f} - \frac{1}{\alpha - J 2\pi f}$$

$$M(f) = \frac{-J 4\pi f}{\alpha^2 + 4\pi^2 f^2}$$

$$F[\operatorname{sgn}(t)] = \lim_{\alpha \to 0} M(f) = \frac{-J 4\pi f}{4\pi^2 f^2}$$

$$= \frac{-1}{\pi f} = \frac{1}{J\pi f}$$

$$\boxed{\operatorname{sgn}(t) \rightleftharpoons \frac{1}{J\pi f}}$$

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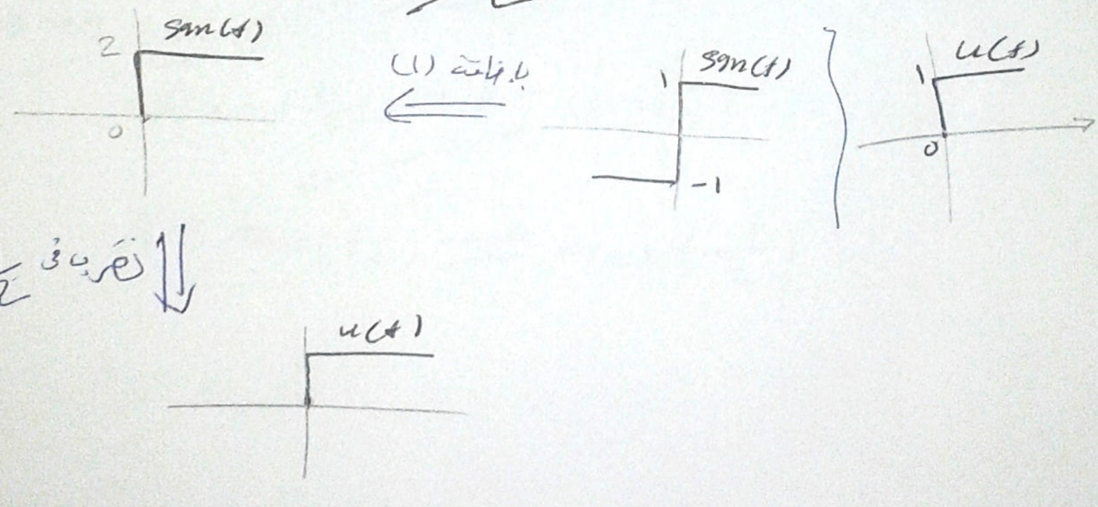
⑨ find F.T for $g(t) = u(t)$

Sol



$$u(t) = \frac{1}{2}\left[\operatorname{sgn}(t) + 1\right]$$

$$F[u(t)] = F\left[\frac{1}{2}(\operatorname{sgn}(t) + 1)\right]$$

$F[u(t)] = F\left[\frac{1}{2}\,\text{sgn}(t) + \frac{1}{2}\right]$

$$= \frac{1}{2}\,\frac{1}{j\pi f} + \frac{1}{2}\,\delta(f)$$

$A \Longrightarrow A\,\delta(f)$

$\frac{1}{2} \Longrightarrow \frac{1}{2}\,\delta(f)$

$1 \Longrightarrow \delta(f)$

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(3) Duality:

If $g(t) \rightleftharpoons G(f)$

$G(t) \rightleftharpoons g(-f)$

Ex:- find FT of $g(t) = A\tau\,\text{sinc}(t\tau)$

Soln

using duality

$$A\,\text{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau\,(\text{sinc}(f\tau))$$

$$A\tau\,\text{sinc}(t\tau) \rightleftharpoons A\,\text{rect}\left(\frac{-f}{\tau}\right)$$

$$A\,\mathrm{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau\,\mathrm{sinc}\,(f\tau)$$

$$A\tau\,\mathrm{sinc}\,(t\tau) \rightleftharpoons A\,\mathrm{rect}\left(\frac{-f}{\tau}\right)$$

center

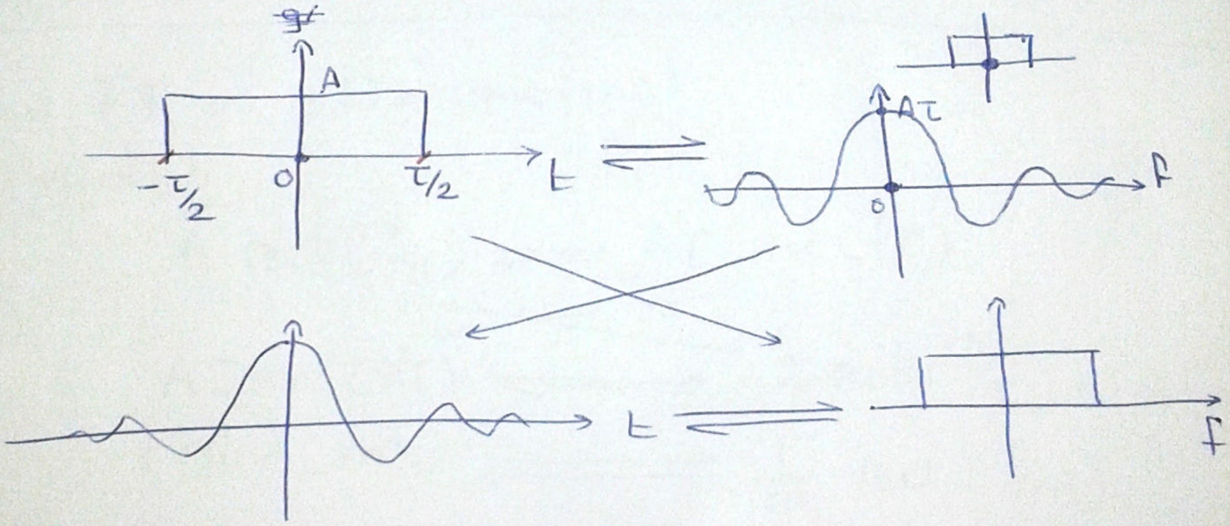
$-f = 0$

مضروب * ±1

بساويه بالصفر

للحصول على الـ Center

$f = 0$



Limited in time → un limited in freq.

والعكس

(13) find F.T. for $g(t) = A \operatorname{sinc}(2\omega t)$.

using duality

$$A \operatorname{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau \operatorname{sinc}(f\tau)$$

$$A\tau \operatorname{sinc}(t\tau) \rightleftharpoons A \operatorname{rect}(f/\tau)$$

$\div\tau$

$$A \operatorname{sinc}(2\omega t) \rightleftharpoons \frac{A}{2\omega} \operatorname{rect}\left(\frac{f}{2\omega}\right)$$

$\tau = 2\omega$

Find F.T. of $g(t) = \operatorname{sinc}(mt)$

using duality

$$A \operatorname{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau \operatorname{sinc}(f\tau)$$

$$A\tau \operatorname{sinc}(t\tau) \rightleftharpoons A \operatorname{rect}(f/\tau)$$

$$1 \cdot \operatorname{Sinc}(mt) \rightleftharpoons \frac{1}{m} \operatorname{rect}(f/m)$$

$\tau = m$

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4 Time-Shift Property

If $g(t) \rightleftharpoons G(f)$

$\theta = \omega t$

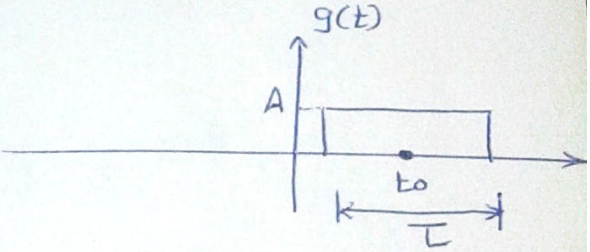
$$g(t \pm t_o) \rightleftharpoons G(f) \cdot e^{\pm J 2\pi f t_o}$$

نفس الاشارة

Find F.T. for $g(t) = A \, \text{rect}\left(\frac{t - t_o}{\tau}\right)$ $\rightarrow = 0$ $t = t_o$ center

$A \downarrow$ Amp.

$\tau \rightarrow$ العرض



using time shift

$$\therefore G(f) = A\tau \, \text{sinc}(f\tau) \cdot e^{-J 2\pi f t_o}$$

$$\because \underbrace{A \, \text{rect}(t/\tau)}_{g(t)} \rightleftharpoons \underbrace{A\tau \, \text{sinc}(f\tau)}_{G(f)}$$

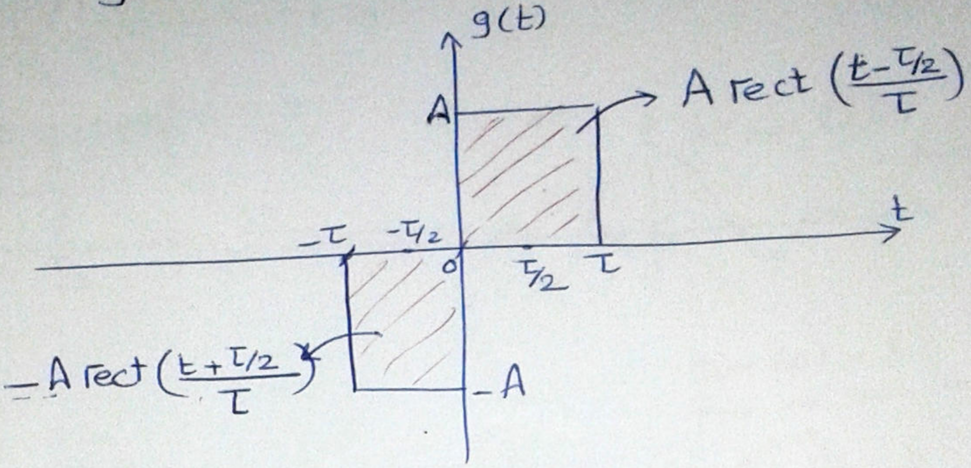
Find F.T. for $g(t) = A \, \text{rect}\left(\frac{t - \tau/2}{\tau}\right)$

using time shift

$$G(f) = A\tau \, \text{sinc}(f\tau) \cdot e^{-J \cancel{2} \pi f \frac{\tau}{\cancel{2}}}$$

$$= A\tau \, \text{sinc}(f\tau) \cdot e^{-J\pi f \tau}$$

Find F.T. of g(t) as shown below



$$g(t) = A\,\text{rect}\left(\frac{t-\tau/2}{\tau}\right) \cancel{*} - A\,\text{rect}\left(\frac{t+\tau/2}{\tau}\right)$$

using Superposition & time shift

$$A\,\text{rect}\left(\frac{t-\tau/2}{\tau}\right) \rightleftharpoons A\tau\,\text{sinc}(f\tau)\cdot e^{-j2\pi f\tau/2}$$

$$A\cdot\text{rect}\left(\frac{t+\tau/2}{\tau}\right) \rightleftharpoons A\tau\,\text{sinc}(f\tau)\cdot e^{+j2\pi f\tau/2}$$

$$G(f) = A\tau\,\text{sinc}(f\tau)\cdot\left[+e^{-j\pi f\tau} - e^{+j\pi f\tau}\right].$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

(5) Frequency - Shift Property

If $\quad g(t) \rightleftharpoons G(f)$

then $\quad g(t) \cdot e^{\pm J 2\pi f_o t} \rightleftharpoons G(f \mp f_o)$

عكس الإشارة

Find F.T. for $g(t) = A \operatorname{rect}\left(\frac{t}{\tau}\right) \cdot e^{-J 2\pi f_o t}$

using frequency - shift

$\because A \operatorname{rect}(t/\tau) \rightleftharpoons A\tau \operatorname{sinc}(f\tau)$

$\therefore G(f) = A\tau \operatorname{sinc}((f+f_o)\tau)$

مهم

Find F.T. $g(t) = A \operatorname{rect}(t/\tau) \cdot \cos(2\pi f_o t)$.

$$g(t) = \frac{1}{2}\left[A \operatorname{rect}\left(\frac{t}{\tau}\right) \cdot \left(e^{J 2\pi f_o t} + e^{-J 2\pi f_o t}\right)\right]$$

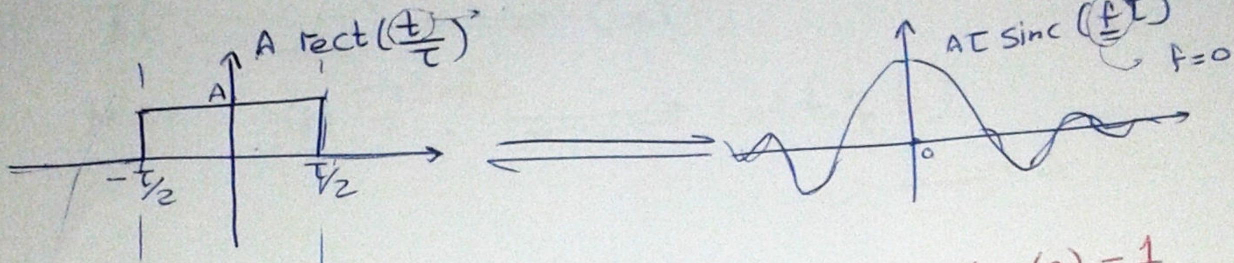
$$= \frac{1}{2}\left[A \operatorname{rect}\left(\frac{t}{\tau}\right) \cdot e^{+J 2\pi f_o t} + A \operatorname{rect}\left(\frac{t}{\tau}\right) \cdot e^{-J 2\pi f_o t}\right]$$

$$\cos\theta = \frac{e^{J\theta} + e^{-J\theta}}{2}$$

$$\sin\theta = \frac{e^{J\theta} - e^{-J\theta}}{2J}$$

using freq. Shift & Linearity

$$G(f) = \frac{1}{2}\left[A\tau\, \mathrm{sinc}(\underbrace{(f-f_0)}_{=0}\tau) + A\tau\, \mathrm{sinc}((f+f_0)\tau) \right].$$



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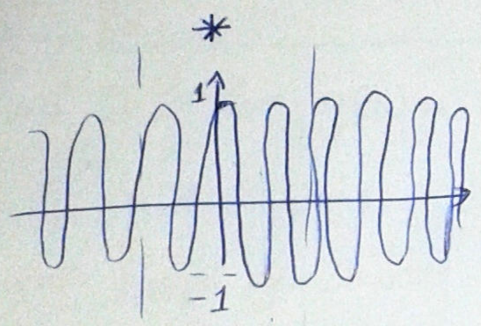


$\mathrm{sinc}(0) = 1$

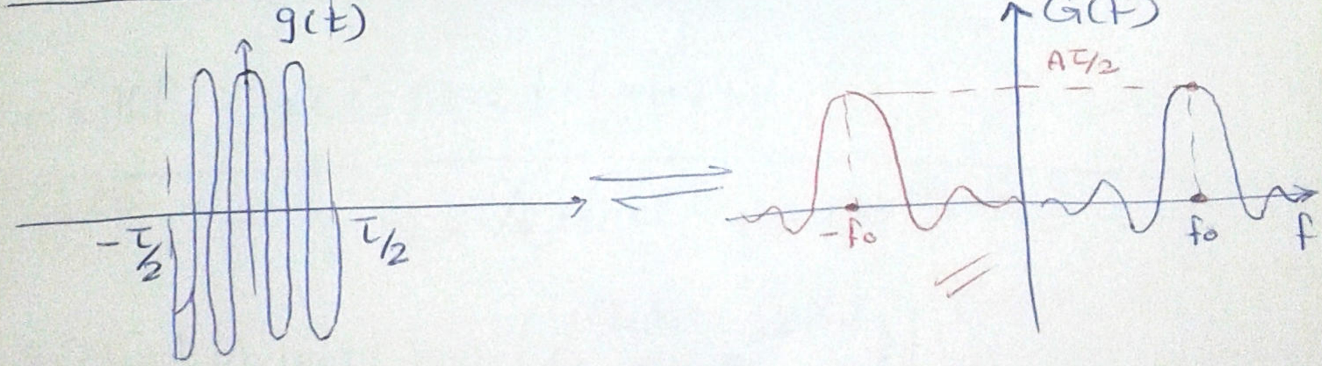
$f=0 \rightarrow$ max. sine

$\mathrm{sinc}(\underset{=0}{x}) = \frac{\sin \pi x}{\pi x}$

$x=0$

$\frac{\sin 0}{0} = 1.$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$



$$\underset{\text{rect}}{m(t)} \cdot \cos(2\pi f_c t) \rightleftharpoons \frac{1}{2}\left[\underset{\text{sinc}}{M(f-f_c)} + M(f+f_c) \right]$$

$$m(t) \cdot \sin(2\pi f_c t) = \frac{1}{2j}\left[m(t) \cdot e^{j\omega_c t} - m(t) \cdot e^{-j\omega_c t} \right]$$

$$\xrightarrow{F.T.} \frac{1}{2j} \cdot \left[M(f-f_c) - M(f+f_c) \right]$$

9

⑥ Area under curve $g(t)$

$$\text{Area} = \int_{-\infty}^{\infty} g(t) \cdot dt$$

$$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi f t} \cdot dt$$

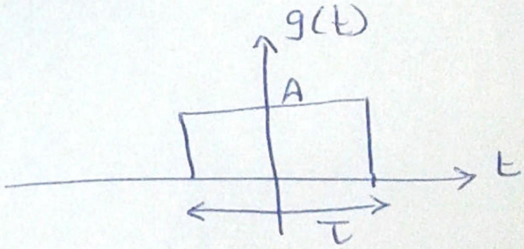
at $f = 0$ $\boxed{\text{Area} = G(0)}$

Find the area under curve $g(t) = A \, \text{rect}(t/\tau)$.

$\because G(f) = A\tau \, \text{sinc}(f\tau)$.

$G(0) = A\tau \, \underbrace{\text{sinc}(0)}_{1}$

$\therefore \text{Area} = G(0) = A\tau$.



Area = $A\tau$.

(7) Area under the Curve G(f)

$$\text{Area} = \int_{-\infty}^{\infty} G(f) \cdot df$$

$$g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{+j2\pi f t} \cdot df$$

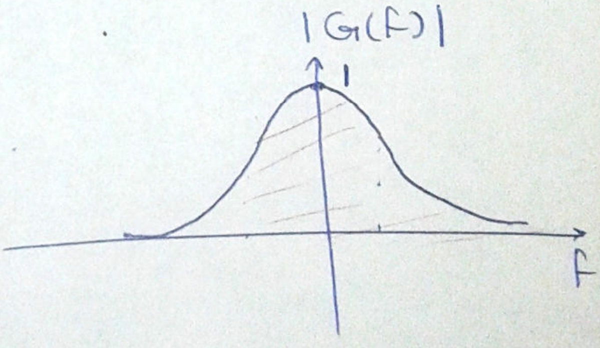
At $t=0$

$$\text{Area} = g(0)$$

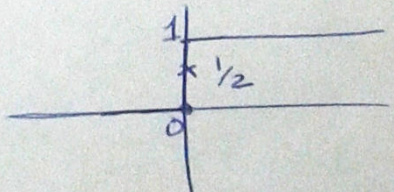
Find Area under $G(f) = \dfrac{1}{1+j2\pi f}$.

$$e^{-t} \cdot u(t) \rightleftharpoons \frac{1}{1+j2\pi f}$$

$$|G(f)| = \frac{1}{\sqrt{1+4\pi^2 f^2}}$$



$$\therefore g(t) = e^{-t} \cdot u(t)$$



$\therefore$ Area under $G(f) = g(0)$

$$= e^{0} \cdot u(0)$$

$$= 1 \cdot \frac{1}{2}$$

$$\therefore \boxed{\text{Area} = \frac{1}{2}}$$